Speed of convergence to stationarity for Möbius monotone Markov chains

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Abstract

The strong stationary duality which is a probabilistic approach to the problem of speed of convergence to stationarity for finite Markov chains was introduced by Diaconis and Fill [2]. This approach involves strong stationary times introduced earlier by Aldous and Diaconis [1] who gave a number of examples showing useful bounds on the total variation distance for convergence to stationarity in cases where other techniques utilizing eigenvalues or coupling were not easily applicable. A *strong stationary time* for a Markov chain (X_n) is a stopping time T for this chain for which X_T has stationary distribution and is independent of T. The tail of T gives bounds on distance to stationarity. Diaconis and Fill [2] constructed an absorbing *dual* Markov chain with its absorption time equal to the strong stationary time T for (X_n).
In general, there is no recipe how to construct dual chains and it is an open problem how to construct a strong stationary time for particular examples of chains. Only a few examples are known. One of them is Diaconis and Fill [2] (Theorem 4.6) when the state space is linearly ordered and the chain has transitions only up and down to the nearest neighbours. In this case, under the assumption of stochastic monotonicity for the time reversed chain, and some conditions on the initial distribution it is possible to construct the dual chain which is a pure birth process on the same state space, and which allows therefore for a relative simple analysis of the speed of convergence of this chain in the total variation distance to the stationary distribution. This topic remains to be very actual. Some recent developments devoted to birth and death chains and duality are contained for example in Fill [4].

We give a construction of duals for arbitrary partially ordered state spaces for chains which are Möbius monotone. This construction is of independent interest because it introduced a new type of monotonicity in place of usually utilized stochastic monotonicity. It turns out that in some cases the resulting dual chain is an analog of the pure birth chain, because its transitions are only upwards.

We apply the results to nonsymmetric random walk on cube and to DNA sequence alignment.

References

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